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Coverage Control Based Effective Jamming Strategy for Wireless Networks

Zhen Kan, John M. Shea, Emily A. Doucette, Jess W. Curtis, and Warren E. Dixon

***Abstract*—A group of mobile jammers is tasked with disrupting the overall communication of a static radio network. The jammers are assumed to have limited jamming capabilities, such that the jamming effect is constrained to a disk area around the jammer. Radios within the jamming zone will be disrupted and the jamming intensity depends on the relative distance between the radio and the jammer. To disrupt the communication network, a dynamic coverage control based jamming strategy is developed, where the jammers coordinate their motion and cooperatively guarantee that every radio in the network is accumulatively disrupted up to a desired jamming level over time. It is further assumed that each jammer has a limited communication capability. Two jammers can only share jamming information when they stay within a certain distance. To ensure consistent jamming coordination, motion control laws are developed for jammers to perform effective jamming while preserving network connectivity among jammers. An appealing feature of the current work is the use of mobile jammers to dynamically disrupt the overall communication network, which enables cooperative jamming over large scale networks by using a limited number of mobile jammers.**

I. Introduction

Wireless communication networks are widely used in commercial, industrial, and military applications. Due to the openness of the wireless medium, such networks are vulnerable to interference, failure, and attack. Various jamming techniques have been developed to disrupt different layers of the protocol stack in wireless networks, such as causing errors in the reception of data in the physical layer [1], blocking transmission of data at the MAC layer [2], or taking advantage of the periodicity of many routing protocols and performing jamming at the network layer [3]. However, most existing results only focus on jamming local communications on one link or at one radio, resulting in degraded global jamming performance, since networks often have multiple paths connecting radios and an alternative route could be used if certain links are jammed.

To disrupt the overall communication of a network, static placement of jammers is considered in [4] and [5] to block network traffic flow by partitioning the network into disconnected subnetworks. The approaches developed in [4] and [5] exploit topological properties of network connectivity and physical locations of the radios, yielding better jamming performance in blocking information exchange between radios. However, since the number of jammers is generally predetermined and often limited, there is no guarantee that the network could be partitioned into several disconnected subnetworks, especially if large scale networks or networks with dense connectivity are considered.

In the present work, a static radio network is considered and a limited number of mobile jammers is tasked with the objective of disrupting the overall communication of the radio network. Each jammer is assumed to have a limited jamming capability, modeled as a disk area around the jammer. Only radios within the jamming zone are disrupted by the jammer, and the jamming intensity depends on the relative distance between the radio and the jammer. Clearly, it is trivial to disrupt a communication network if a sufficient number of jammers are available such that the union of jamming zones of all jammers can completely cover the entire network. If fewer jammers are provided, but still sufficient to partition the network into disconnected subnetworks as discussed in the results of [4] and [5], communication disruption is still guaranteed. To relax the constraints on the number of jammers, in this work, mobile jammers are considered, where the number of jammers is not large enough to partition the communication network as in [4] and [5]. Therefore, it is desirable to take advantage of the jammer mobility and develop a cooperative motion strategy to dynamically disrupt the the overall communication in the wireless network.

Coverage control is a type of cooperative control for a multi-agent system to continuously monitor an area of interest. Coverage control typically consists of either static or dynamic coverage control. Static coverage control generally addresses the problems of optimal placement of sensors to cover a region of interest (cf. [6]–[8], to name a few), while dynamic coverage control (cf. [9]–[12]) focuses on searching an area sufficiently well over time. Particularly, in dynamic coverage control, mobile sensors are tasked to perform effective coverage, where every point in the area of interest is ensured to be visited for a sufficient amount of time. Inspired by the effective coverage developed in [9]–[12], the jamming strategy considered in the current work is to disrupt the overall communication by ensuring every radio in the network is accumulatively disrupted by the group of mobile

Zhen Kan and Warren E. Dixon are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL, USA. Email: {kanzhen0322, wdixon}@ufl.edu. John M. Shea is with the Department of Electrical and Computer Engineering, University of Florida, Gainesville, USA. Email: jshea@ece.ufl.edu. Emily A. Doucette and Jess W. Curtis are with the Munitions Directorate, Air Force Research Laboratory, Eglin AFB, FL, USA. Email: {emily.doucette, jess.curtis}@us.af.mil.

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jammers up to a preset jamming level over time. Since the radios form a connected network, from the point of view of graph topology, certain radios have more important roles than others in terms of communication and information relay. For instance, nodes with the most connections in the network or nodes whose removal can lead to disconnected networks are obviously more critical than other nodes. Hence, each radio is associated with a different jamming level that indicates how much jamming effort should be imposed, and motion control laws are developed for the jammers to cooperatively ensure every radio is disrupted up to the desired jamming level. It is assumed that each jammer has a limited communication capability. Two jammers can only communicate when they are within a certain distance. The motion of jammers is then further constrained to preserve network connectivity among the jammers while performing effective jamming. In [13] and [14], distributed control algorithms are develeoped to maximize area coverage by a mobile robot network while ensuring reliable communication between the robots. Other representative results on motion planning for mobile networks with preservation of network connectivity include [15]–[21].

Compared to the results such as [1]–[3], our jamming strategy focuses on disrupting the overall communication in a network, rather than jamming a single communication link. As discussed in [4] and [5], the network partition based jamming strategy requires expensive computation effort and is not always guaranteed to partition the network by a given number of jammers, especially for large scale networks. In contrast, the use of mobile jammers in the current work allows disruption of a larger network by taking advantage of its mobility. Moreover, the developed jamming strategy can dynamically disrupt the overall communication network, making it difficult for the network to respond and recover from jamming attacks.

II. Problem Formulation

Consider M static radios distributed in a two-dimensional compact workspace $\mathcal{W}$ and let $p_j \in \mathbb{R}^2$ denote the position of radio $j \in \{1, \ldots M\}$ within $\mathcal{W}$. The radios are assumed to be homogeneous with equal transmit power and omnidirectional antennas, which form a connected communication network modeled by an undirected graph $\mathcal{G}_T = (\mathcal{V}_T, \mathcal{E}_T)$, where the vertex set $\mathcal{V}_T$ represents the radios and the edge set $\mathcal{E}_T$ represents wireless communication links between radios.

A group of N mobile jammers is tasked to disrupt the communication of the network $\mathcal{G}_T$. The jammers are assumed to move according to the single-integrator kinematics

$$\dot{x}_i = u_i, \ i = 1, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^2$ denotes the position of jammer i, and $u_i \in \mathbb{R}^2$ represents its control input.

It is assumed that jammers have complete knowledge of the radios' positions. Jammers can disrupt wireless communication in various ways such as reducing the signal-to-interference ratio at the receiver or overloading the front end of a receiver. In the present work, each jammer is assumed to have a limited jamming zone $\mathcal{S}_i$, encoded as a disk area with radius $r \in \mathbb{R}^+$ centered at jammer i. Since radio signals generally follow an exponential path-loss model, inspired by [9], the distance based jamming intensity is characterized by[1]

$$J_i\left(x_i\left(t\right), p_j\right) = \begin{cases} \frac{M_p}{r^4}\left(s_{ij} - r^2\right)^2, & s_{ij} \leq r^2, \\ 0, & s_{ij} > r^2, \end{cases} \tag{2}$$

where $s_{ij}\left(t\right) \triangleq \left\|x_i\left(t\right) - p_j\right\|^2$, and $M_p \in \mathbb{R}^+$ is a peak jamming capability.

Let $\operatorname{int}\left(\mathcal{S}_i\right)$, $\partial\mathcal{S}_i$, and $\operatorname{ext}\left(\mathcal{S}_i\right)$ denote the interior, boundary, and exterior of jamming zone $\mathcal{S}_i$, respectively. Clearly, $J_i\left(x_i, p_j\right) > 0$ if $p_j \in \operatorname{int}\left(\mathcal{S}_i\right)$ and $J_i\left(x_i, p_j\right) = 0$ if $p_j \in \partial\mathcal{S}_i \cup \operatorname{ext}\left(\mathcal{S}_i\right)$. The jamming model in (2) indicates that the jamming intensity achieves its peak value when jammer i coincides with radio j and monotonically degrades as jammer i moves away from radio j, which indicates that the jamming becomes less effective as radios move closer to the boundary of the jamming zone. Note that the subsequent development is not limited to the particular jamming model proposed in (2). Other functions such as sigmoid functions or hyperbolic tangent functions with appropriate modification can also serve as a qualified jamming model.

Due to the limited jamming capability, the jammers are required to cooperatively disrupt the overall communication of the target network $\mathcal{G}_T$. The interaction among jammers (i.e., communication and information exchange) is modeled by an undirected graph $\mathcal{G}\left(t\right) = \left(\mathcal{V}, \mathcal{E}\left(t\right)\right)$, where $\mathcal{V}$ represents the set of jammers and $\mathcal{E}\left(t\right)$ represents the set of communication links between jammers. It is further assumed that each jammer has a limited communication capability encoded by a disk area with radius R, which implies that two jammers can only exchange information within a distance of R. Hence, the edge $(i, k) \in \mathcal{E}$ between jammer i and k is established only when their relative distance $\left\|x_i - x_k\right\|_2$ is less than R. Let $\mathcal{N}_i = \left\{k \in \mathcal{V} | \ \ (i, k) \in \mathcal{E}\left(t\right)\right\}$ denote the neighbors of jammer i. The graph $\mathcal{G}$ is connected if there exists a path connecting any two nodes in the graph.

An example problem scenario is illustrated in Fig. 1. A network of 100 radios is deployed in a two-dimensional plane, forming a connected communication network where the dots represent the radios and the solid lines represent the inter-radio wireless communication. The mobile jammers are denoted by triangles and the shaded disks indicate their limited jamming zones. Radios within the jamming zones are under jamming attack and the disrupted communication is indicated by dashed line.

The main objective in the present work is to develop cooperative motion control laws for the jammers to guarantee that every radio in the network is accumulatively disrupted up to a desired jamming level over time. The jamming level will be defined and discussed in detail in the subsequent

[1]The current work is based on the simplified distance-based jamming model in (2). Additional work will consider more realistic communication (e.g., path loss, shadowing, and multi-path fading) in jamming models based on the results developed in [18]–[21].

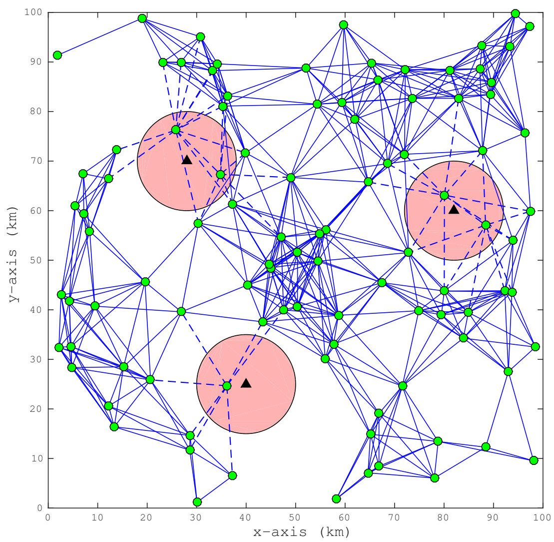

Figure 1. An example problem scenario, where three mobile jammers, denoted by triangles, are tasked to disrupt the communication of a network of 100 radios. The dots represent the radios and the solid lines indicates the inter-radio wireless communication links. The shaded disks represent the limited jamming zones. Radios within the jamming zones are under jamming attack and the disrupted communication is indicated by dashed lines.

section. It is assumed that the initial graph $\mathcal{G}(0)$ is connected. To ensure information exchange among the jammers, the jammers motion are further constrained to preserve network connectivity of $\mathcal{G}$ while performing effective jamming.

III. Effective Jamming Strategy

The concept of effective coverage is originally developed in the works of [9] and [10] for sensing a compact region. Inspired by coverage control, the effective jamming strategy in the current work is to drive the mobile jammers to ensure every radio in the network $\mathcal{G}_T$ is accumulatively disrupted up to a desired level over time. Let $\phi_i(t; 0, x_i(0))$ denote a trajectory of jammer i within a time interval $[0, t]$ under the controller u_i in (1) with the initial condition $x_i(0) \in \mathbb{R}^2$. The effective jamming achieved by jammer $i \in \mathcal{V}$ over radio $j \in \mathcal{V}_T$ along the trajectory $\phi_i(t; 0, x_i(0))$ is defined as

$$Q_{i,j}(\phi_i, p_j) \triangleq \int_0^t J_i(x_i(\tau), p_j)\, d\tau, \tag{3}$$

which quantifies the accumulated jamming imposed by jammer i on radio j over $[0, t]$.

Based on the individual jamming on radio j designed in (3), the jamming on radio j by the entire group of jammers is defined as

$$Q_{\mathcal{V},j}(\phi_1, \ldots, \phi_N, p_j) \triangleq \int_0^t \sum_{i\in\mathcal{V}} J_i(x_i(\tau), p_j)\, d\tau, \tag{4}$$

where $Q_{\mathcal{V},j}$ is a function of the trajectories of all jammers over $[0, t]$.

Due to different roles of radios in the network $\mathcal{G}_T$, certain radios are more important than others in terms of communication and information relay. For instance, from the view of graph topology, the set of nodes whose removal can lead to disconnected networks are obviously more critical than other nodes, since the communication within $\mathcal{G}_T$ can be maximally disrupted by jamming such set of nodes, resulting a partitioned communication network. Hence, according to its importance, each radio j is associated with a desired jamming level $Q_j^* \in \mathbb{R}^+$ that indicates how much jamming effort should be imposed on it. The objective of effective jamming over radio j is achieved if $Q_{\mathcal{V},j} \geq Q_j^*$.

To maximally disrupt the communication, critical radios are supposed to have larger values of jamming level (e.g., larger $Q_{\mathcal{V},j}$ for critical radio j), which indicates that more jamming effort should be spent on critical radios. In this initial work, the desired jamming levels Q_j^* for all $j \in \mathcal{V}_T$ are simply assumed known and predetermined. Ongoing work is to determine the radio jamming level based on the trade-off between overall jamming performance and control effort of jammers. For instance, graph topology properties in [22] could be exploited to determine a subset of nodes whose removal can maximally impact the communication of a given network.

Consider a non-negative and twice differentiable penalty function $h(w) = (\max\{0, w\})^3$ with its first derivative $h'(w) = \frac{dh}{dw} = 3(\max\{0, w\})^2$ and second derivative $h''(w) = 6\max\{0, w\}$. The objective of disrupting the overall communication of $\mathcal{G}_T$ is then formulated as

$$e(t) = \sum_{j\in\mathcal{V}_T} h\left(Q_j^* - Q_{\mathcal{V},j}\right). \tag{5}$$

Clearly, $e(t) = 0$ in (5) indicates mission completion since every radio j is accumulatively disrupted to its desired level Q_j^* over $[0, t]$ (i.e., $Q_{\mathcal{V},j}(t) \geq Q_j^*$ for $\forall j \in \mathcal{V}_T$). Note that $h(w) = 0$ when $w \leq 0$, which also implies in (5) that no penalty is incurred if $Q_{\mathcal{V},j} > Q_j^*$ (i.e., more jamming effort is spent on radio j), since it results in more jamming over the communication.

Based on the Leibniz integral rule, the time derivative of $Q_{i,j}$ and $Q_{\mathcal{V},j}$ is obtained as

$$\dot{Q}_{ij} = J_i(x_i(t), p_j)$$

from (3) and

$$\dot{Q}_{\mathcal{V},j} = \sum_{i\in\mathcal{V}} J_i(x_i(t), p_j) \tag{6}$$

from (4). Using (6), the time derivative of $e(t)$ is

$$\dot{e}(t) = -\sum_{j\in\mathcal{V}_T} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)\left(\sum_{i\in\mathcal{V}} J_i(x_i(t), p_j)\right). \tag{7}$$

Since $h(w) = h'(w) = 0$ when $w \leq 0$ and $h(w)$ and $h'(w)$ are all positive when $w > 0$, the term $\sum_{j\in\mathcal{V}_T} h'\left(Q_j^* - Q_{\mathcal{V},j}\right) \to 0$ in (7) indicates that

$\sum_{j\in\mathcal{V}_T} h\left(Q_j^* - Q_{\mathcal{V},j}\right) \to 0$ (i.e., $e(t) \to 0$). However, $\dot{e}(t) \to 0$ does not indicate that $e(t) \to 0$ since $\dot{e}(t)$ could be zero when $\sum_{i\in\mathcal{V}} J_i(x_i, p_j) = 0$ while $h'\left(Q_j^* - Q_{\mathcal{V},j}\right) \neq 0$, and this condition may occur, for instance, when radio j does not fall in the jamming zone of any jammer $i \in \mathcal{V}$ from (2).

Inspired by [11], a variant of $\dot{e}(t)$ is used in the subsequent development.

Lemma 1. *Let a variant of $\dot{e}(t)$ be defined as*

$$V_c(\phi(t)) \triangleq \sum_{j\in\mathcal{V}_T} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)\left(\sum_{i\in\mathcal{V}} J_i(x_i, p_j) + \alpha\right) \tag{8}$$

where $\phi(t) = \{\phi_i\}$, $i \in \mathcal{V}$, representing the set of trajectories of all jammers, and α is a positive constant. The designed $V_c(\phi(t)) \geq 0$ and $V_c(\phi(t)) = 0$ if and only if the objective of effective jamming over $\mathcal{G}_T$ is achieved (i.e., $Q_{\mathcal{V},j} \geq Q_j^*$ for $\forall j \in \mathcal{V}_T$).

Proof: The term $\sum_{i\in\mathcal{V}} J_i(x_i, p_j) + \alpha$ in (8) is strictly positive, since $\alpha > 0$ and $J_i \geq 0$ from (2). Hence, $V_c(\phi(t)) \geq 0$, since $h'(\cdot)$ is non-negative. When effective jamming is achieved, $Q_{\mathcal{V},j} \geq Q_j^*$ for $\forall j \in \mathcal{V}_T$, which implies that $h'\left(Q_j^* - Q_{\mathcal{V},j}\right) = 0$ for $\forall j \in \mathcal{V}_T$ and thus $V_c(\phi(t)) = 0$. The converse is easily verified, since $V_c(\phi(t)) = 0$ implies that $h'\left(Q_j^* - Q_{\mathcal{V},j}\right) = 0$ for $\forall j \in \mathcal{V}_T$ since $\sum_{i\in\mathcal{V}} J_i(x_i, p_j) + \alpha$ is strictly positive. ■

Lemma 1 shows that V_c in (8) can be treated as an error function to indicate whether the objective of effective jamming is achieved. It is worth pointing out that V_c itself dose not contain $\dot{x}_i$. Since the time derivative of V_c contains $\dot{x}_i$, the control design for $\dot{x}_i$ in the the subsequent development is based on the insights of Lyapunov based convergence analysis. To develop the motion control laws for the jammers, the following assumptions are required.

Assumption 1. The jamming information $Q_{\mathcal{V},j}$, $\forall j \in \mathcal{V}_T$, is available to each jammer $i \in \mathcal{V}$, as long as $\mathcal{G}(t)$ remains connected.

Assumption 2. The jammers are restricted within the compact workspace $\mathcal{W}$ and, at any time instant, at least one radio falls in the jamming zone of a jammer during effective jamming (i.e., $\sum_{i\in\mathcal{V}} J_i(x_i, p_j) \neq 0$ for at least one radio j).

To coordinate the motion of jammers to disrupt the network $\mathcal{G}_T$, Assumption 1 indicates that jammers are able to communicate and share information on how the radio j is disrupted by all jammers. Section IV will discuss how the motion of jammers will be constrained to ensure the underlying graph $\mathcal{G}(t)$ is connected when performing effective jamming. Assumption 2 indicates that jammers will not move out of $\mathcal{W}$ or move into a certain configuration that will cause $\sum_{j\in\mathcal{V}_T}\sum_{i\in\mathcal{V}} J_i(x_i, p_j) = 0$. Note that Assumption 2 could be relaxed, especially if a densely populated radio network or jammers with large jamming radius are considered.

Theorem 1. *Provided that Assumption 1 and Assumption 2 hold, the following motion control law*

$$\bar{u}_i = -\sum_{p_j\in\mathrm{int}(\mathcal{S}_i)} \frac{dJ_i}{ds_{ij}} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)(x_i - p_j), \tag{9}$$

ensures every jammer i with kinematics $\dot{x}_i = \bar{u}_i$ converges to the set

$$\Psi \triangleq \left\{ x_i \in \mathbb{R}^2, \forall i \in \mathcal{V} \middle| Q_{\mathcal{V},j} \geq Q_j^*, \forall p_j \in \mathrm{int}(\mathcal{S}_i)\right\}. \tag{10}$$

Proof: Since $V_c(\phi(t)) \geq 0$ and $V_c(\phi(t)) = 0$ only when the effective jamming is accomplished based on Lemma 1, consider $V_c(\phi(t))$ as a Lyapunov candidate.

Based on the definition of J_i in (2),

$$\frac{dJ_i}{dt} = \frac{dJ_i}{ds_{ij}} 2(x_i - p_j)^T \dot{x}_i \tag{11}$$

where

$$\frac{dJ_i}{ds_{ij}} = \begin{cases} \frac{2M_p}{r^4}\left(s_{ij} - r^2\right), & s_{ij} < r^2, \\ 0, & s_{ij} \geq r^2. \end{cases} \tag{12}$$

Taking time derivative of $V_c(\phi(t))$ and using (6) and (11) yields

$$\begin{aligned}\dot{V}_c = &-\sum_{j\in\mathcal{V}_T} h''\left(Q_j^* - Q_{\mathcal{V},j}\right)\sum_{i\in\mathcal{V}} J_i\left(\sum_{i\in\mathcal{V}} J_i + \alpha\right) \\ &+ \sum_{j\in\mathcal{V}_T} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)\left(\sum_{i\in\mathcal{V}} \frac{dJ_i}{ds_{ij}} 2(x_i - p_j)^T \dot{x}_i\right),\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\dot{V}_c = &-\sum_{j\in\mathcal{V}_T} h''\left(Q_j^* - Q_{\mathcal{V},j}\right)\sum_{i\in\mathcal{V}} J_i\left(\sum_{i\in\mathcal{V}} J_i + \alpha\right) \\ &+ 2\sum_{i\in\mathcal{V}}\left(\sum_{j\in\mathcal{V}_T} \frac{dJ_i}{ds_{ij}} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)(x_i - p_j)^T\right)\dot{x}_i.\end{aligned} \tag{13}$$

Since $\frac{dJ_i}{ds_{ij}} = 0$ for $p_j \in \partial\mathcal{S}_i \cup \mathrm{ext}(\mathcal{S}_i)$, $j \in \mathcal{V}_T$, based on (12), (13) can be simplified as

$$\begin{aligned}\dot{V}_c = &-\sum_{j\in\mathcal{V}_T} h''\left(Q_j^* - Q_{\mathcal{V},j}\right)\sum_{i\in\mathcal{V}} J_i\left(\sum_{i\in\mathcal{V}} J_i + \alpha\right) \\ &+ 2\sum_{i\in\mathcal{V}}\left(\sum_{p_j\in\mathrm{int}(\mathcal{S}_i)} \frac{dJ_i}{ds_{ij}} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)(x_i - p_j)^T\right)\dot{x}_i.\end{aligned} \tag{14}$$

Substituting the controller (9) into (14) yields

$$\begin{aligned}\dot{V}_c = &-\sum_{j\in\mathcal{V}_T} h''\left(Q_j^* - Q_{\mathcal{V},j}\right)\sum_{i\in\mathcal{V}} J_i\sum_{i\in\mathcal{V}}(J_i + \alpha) \\ &- 2\sum_{i\in\mathcal{V}}\left\|\sum_{p_j\in\mathrm{int}(\mathcal{S}_i)} \frac{dJ_i}{ds_{ij}} h'\left(Q_j^* - Q_{\mathcal{V},j}\right)(x_i - p_j)\right\|^2,\end{aligned} \tag{15}$$

which indicates that $\dot{V}_c \leq 0$, since $h''(\cdot)$ and J_i are all non-negative functions.

Based on Assumption 2, there exists at least one radio disrupted by a jammer at any time instant. For any radio j that falls in the jamming zone of a jammer, it is true that $\sum_{i\in\mathcal{V}} J_i(x_i, p_j) > 0$. If $\dot{V}_c = 0$, it must have $h''\left(Q_j^* - Q_{\mathcal{V},j}\right) = 0$ from the first line in (15), which indicates all radios j with $p_j \in \operatorname{int}(\mathcal{S}_i), \forall i \in \mathcal{V}$, will be disrupted to a desired jamming level, i.e., $Q_{\mathcal{V},j} \geq Q_j^*$. The second line in (15) will also be zero, since $h''\left(Q_j^* - Q_{\mathcal{V},j}\right) = h'\left(Q_j^* - Q_{\mathcal{V},j}\right) = 0$ if $Q_{\mathcal{V},j} \geq Q_j^*$, .which indicates that the largest invariant set for $\dot{V}_c = 0$ is

$$\Psi = \left\{ x_i, \forall i \in \mathcal{V} \middle| Q_{\mathcal{V},j} \geq Q_j^*, \forall p_j \in \operatorname{int}(\mathcal{S}_i) \right\}.$$

Since $\frac{dJ_i}{ds_{ij}}$ and $Q_{\mathcal{V},j}$ in (9) are functions of jammers trajectories only, $\dot{x}_i = \bar{u}_i,\ i = 1, \ldots, N$, are autonomous systems. LaSalle's invariance principle in [23] can then be invoked to conclude that the system will converge to the set Ψ. ■

IV. Connectivity Maintenance

The cooperative jamming strategy designed in (9) is based on Assumption 1, which requires a connected communication network $\mathcal{G}(t)$ so that jammers can exchange the jamming information $Q_{\mathcal{V},j},\ \forall j \in \mathcal{V}_T$. However, due to limited communication capabilities, the motion of jammers can lead to a disconnected network $\mathcal{G}(t)$, resulting in failure of sharing jamming information. In addition to performing effective jamming, the motion of jammers is further constrained to preserve network connectivity in this section.

Consider an escape region for each jammer i, defined as the outer ring of the communication area with radius r, $R - \delta < r < R$, where $\delta \in \mathbb{R}^+$ is a predetermined buffer distance. Edges formed with any jammer $k \in \mathcal{N}_i$ in the escape region are in danger of breaking. Inspired by our earlier results in [15] and [16], to ensure the existing link $(i,k) \in \mathcal{E}(t)$ is connected, consider a penalty function $C_{ik}(x_i, x_k)$

$$C_{ik}(x_i, x_k) \triangleq \left(\min\left\{0, \frac{d_{ik} - (R-\delta)^2}{d_{ik} - R^2}\right\}\right)^2, \qquad (16)$$

where $d_{ik} \triangleq \|x_i - x_k\|^2$. In (16), the non-negative function $C_{ik}(x_i, x_k) \to \infty$ as $d_{ik} \to R^2$ (i.e., the edge (i,k) is about to break) and $C_{ik}(x_i, x_k)$ monotonically decreases to zero as their inter-distance d_{ik} decreases to $(R-\delta)^2$.

The partial derivative of $C_{ik}(x_i, x_k)$ with respect to x_i is then given by

$$\frac{\partial C_{ik}}{\partial x_i} = \begin{cases} 0, & d_{ik} \leq (R-\delta)^2 \\ \Xi, & (R-\delta)^2 < d_{ik} < R^2 \\ \text{undefined}, & d_{ik} = R^2. \end{cases}, \qquad (17)$$

where

$$\Xi \triangleq 4\min\left\{0, \frac{d_{ik} - (R-\delta)^2}{d_{ik} - R^2}\right\} \frac{\left((R-\delta)^2 - R^2\right)(x_i - x_k)^T}{\left(d_{ik} - R^2\right)^2}.$$

The following theorem will show that if two jammers i and k are connected initially (i.e., $(i,k) \in \mathcal{E}(0)$), they will remain connected, i.e., $d_{ik} < R^2$, which ensures that the undefined $\frac{\partial C_{ik}}{\partial x_i}$ at $d_{ik} = R^2$ will not introduce any discontinuity to the system.

Theorem 2. *Let Φ denote*

$$\Phi \triangleq \left\{ \mathbf{x} \in \mathbb{R}^{2N} \middle| d_{ik} < (R-\delta)^2, \forall (i,k) \in \mathcal{E} \right\},$$

where $\mathbf{x} = \left[x_1^T, \ldots, x_N^T\right]^T$. Jammers with kinematics (1) are ensured to converge to $\Psi \cap \Phi$ by following the control law

$$u_i = \bar{u}_i + u_i^*, \qquad (18)$$

where $\bar{u}_i$ is the control law for effective jamming designed in (9) and u_i^* is defined as

$$u_i^* = -\sum_{k\in\mathcal{N}_i} \frac{\partial C_{ik}}{\partial x_i} \qquad (19)$$

to preserve network connectivity.

Proof: Consider a Lyapunov function candidate

$$V = V_c(\phi(t)) + V_p(\mathbf{x}(t)),$$

where V_c is from (8) and V_p is defined as

$$V_p(\mathbf{x}(t)) \triangleq \sum_{i\in\mathcal{V}} \sum_{k\in\mathcal{N}_i} C_{ik}(x_i, x_k). \qquad (20)$$

Taking time derivative of V and using (14) and (18) yields

$$\dot{V} = \dot{V}_c + \dot{V}_p, \qquad (21)$$

where $\dot{V}_c$ in (21) is

$$\begin{aligned} \dot{V}_c = &-\sum_{j\in\mathcal{V}_T} h''\left(Q_j^* - Q_{\mathcal{V},j}\right) \sum_{i\in\mathcal{V}} J_i \left(\sum_{i\in\mathcal{V}} J_i + \alpha\right) \\ &- 2\sum_{i\in\mathcal{V}} \bar{u}_i^T\left(\bar{u}_i + u_i^*\right), \end{aligned} \qquad (22)$$

and $\dot{V}_p$ in (21) is computed from (20) as

$$\begin{aligned} \dot{V}_p &= \sum_{i\in\mathcal{V}} \sum_{k\in\mathcal{N}_i} \left(\frac{\partial C_{ik}}{\partial x_i} u_i + \frac{\partial C_{ik}}{\partial x_k} u_k\right) \\ &= \sum_{i\in\mathcal{V}} \sum_{k\in\mathcal{N}_i} \left(\frac{\partial C_{ik}}{\partial x_i} u_i + \frac{\partial C_{ki}}{\partial x_k} u_k\right), \end{aligned} \qquad (23)$$

where the fact that $C_{ik} = C_{ki}$ from (16) is used. Note that, for undirected graphs, it is always true that

$$\sum_{i\in\mathcal{V}} \sum_{k\in\mathcal{N}_i} \frac{\partial C_{ki}}{\partial x_k} u_k = \sum_{i\in\mathcal{V}} \sum_{k\in\mathcal{N}_i} \frac{\partial C_{ik}}{\partial x_i} u_i. \qquad (24)$$

Using (24) and $\frac{\partial C_{ik}}{\partial x_i} = -\frac{\partial C_{ik}}{\partial x_k}$ from (17), $\dot{V}_p$ in (23) can be further simplified as

$$\begin{aligned} \dot{V}_p &= \sum_{i\in\mathcal{V}} \left(2\sum_{k\in\mathcal{N}_i} \frac{\partial C_{ik}}{\partial x_i}\right) u_i \\ &= -\sum_{i\in\mathcal{V}} 2\left(u_i^*\right)^T\left(\bar{u}_i + u_i^*\right). \end{aligned} \qquad (25)$$

Substituting (22) and (25) into $\dot{V}$ yields

$$\begin{aligned}\dot{V}=&-\sum_{j\in\mathcal{V}_T}h''\left(Q_j^*-Q_{\mathcal{V},j}\right)\sum_{i\in\mathcal{V}}J_i\left(\sum_{i\in\mathcal{V}}J_i+\alpha\right)\\&-\sum_{i\in\mathcal{V}}2\left(\bar{u}_i+u_i^*\right)^T\left(\bar{u}_i+u_i^*\right),\end{aligned}\tag{26}$$

which indicates that $\dot{V}\leq 0$. Since $C_{ik}\to\infty$, if an edge (i,k) is disconnected, the bounded V indicates that every existing edge in $\mathcal{G}(t)$ is preserved.

Following similar analysis in Theorem 1, the first line in (26) is zero only when all jammers will converge to Ψ, which also indicates that $\bar{u}_i=0$ for $\forall i\in\mathcal{V}$. To ensure $\dot{V}=0$ in (26), note that $u_i^*=0,\forall i\in\mathcal{V}$, if all jammers are in the set Φ, since $\frac{\partial C_{ik}}{\partial x_i}=0$ from (17). Hence, all jammers will converge to the set $\Psi\cap\Phi$, which indicates the network connectivity is preserved while every radio within the jamming zone of jammers are ensured to be effectively jammed. ■

Remark 1. Theorem 2 indicates that, under the controller u_i in (18), the communication of $\mathcal{G}_T$ will be disrupted until, for any jammer i, every radio j within its jamming zone $\mathcal{S}_i$ (i.e., $p_j\in\operatorname{int}(\mathcal{S}_i)$) has been disrupted up to a desired jamming level with $Q_{\mathcal{V},j}\geq Q_j^*$ (i.e., convergence to the set Ψ). However, it does not guarantee that every radio in the network $\mathcal{G}_T$ will be effectively disrupted, since the controller u_i only ensures effective jamming of radios within $\mathcal{S}_i,\forall i\in\mathcal{V}$. In addition, the controller u_i vanishes when every radio j with $p_j\in\operatorname{int}(\mathcal{S}_i)$ has been effectively jammed and Φ holds, which indicates that jammer i stops moving even if there exists a radio k with $p_k\in\operatorname{ext}(\mathcal{S}_i)$ that has not been effectively jammed. To ensure effective jamming of the entire network $\mathcal{G}_T$, following similar ideas in [10], a perturbation control law could be applied to drive jammer i to its nearest radio that has not been effectively jammed, if jammer i stops moving and the entire network $\mathcal{G}_T$ has not been effectively jammed. Once the radio j with $Q_{\mathcal{V},j}<Q_j^*$ falls in $\mathcal{S}_i$, the jamming control (18) will be nonzero and could be applied again to perform effective jamming. Repeating this procedure, if necessary, can eventually ensure that every radio $j\in\mathcal{V}_T$ has $Q_{\mathcal{V},j}\geq Q_j^*$.

V. Conclusion

A dynamic coverage control based effective jamming strategy is developed in this work to disrupt the overall communication of a static radio network, where the mobile jammers coordinate their motion and cooperatively guarantee that every radio in the network is accumulatively disrupted up to a desired jamming level over time. In the current paper, the radio jamming level is predetermined. Future work will focus on dynamically determining the jamming level based on graph topology properties. For instance, the network partition based strategy in our previous works of [4] and [5] could be modified to indicate the desired jamming levels for different nodes in the network, since nodes that are critical to network connectivity demands more jamming effort on them.

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